1. Method of Combination of Variables (similarity solutions)- semi-infinite regions such that initial condition and boundary condition at infinity may be combined into single new boundary condition
2. Method of Separation of Variables- partial differential equation split up into 2+ ordinary differential equations. Solution is infinite sum of products of solutions of ordinary differential equations.
3. Method of Sinusoidal Response- not covered in this class

Example 4.1-1: Semi-infinite body of liquid with constant density and viscosity bounded below by horizontal surface (xz plane). Initially fluid and solid are at rest. At time t=0, solid surface is set in motion in positive x direction with velocity v0. Find velocity vx as function of y and t. No pressure gradient or gravity force in x direction. Flow is laminar.

Vx = vx(y,t), vy = 0, vz = 0

Table B4: equation of continuity satisfied

Table B5: ∂vX/∂t = *v* ∂2vx/∂y2

*v* = μ/ρ

I.C: t <= 0, vx = 0 for all y

B.C. 1: y = 0, vX = v0 for t > 0

B.C. 2: y = inf, vx = 0 for t > 0

Dimensionless velocity: Փ = vx/v0

∂Փ/∂t = *v* ∂2Փ/∂y2

Փ(y,0) = 0, Փ(0,t) = 1, Փ(inf.,t) = 0

Փ = Փ(y,t;*v*), y, t, *v* must be in dimensionless combination: y/sqrt(*v*t)

Փ = Փ(η) where η = y/sqrt(4*v*t) (4 included to make final result neater)

Convert derivatives with respect to η

∂Փ/∂t =∂Փ/∂η \* ∂η/∂t = -1/2 \* η/t \* ∂Փ/∂η

∂Փ/∂y = ∂Փ/∂η \* ∂η/∂y = ∂Փ/∂η \* 1/sqrt(4*v*t)

∂2Փ/∂y2 = ∂2Փ/∂η2 \* 1/4*v*t

Substitute into dimensionless derivative:

∂2Փ/∂η2 + 2η \* ∂Փ/∂η = 0

New Boundary Conditions:

B.C.1: η = 0, Փ = 1

B.C.2: η = inf., Փ = 0

Obtain first order separable equation)

∂Փ/∂η = Ψ = C1e-η2

Integrate to get Փ

Փ = C1 ∫0η e-nbar2 dnbar+ C2 (ηbar distinguished from η)

Փ(η) = 1 - erf η (ratio of integrals = error function)

Rewrite in original variables

vx(y,t)/v0= 1 - erf y/sqrt(4*v*t) = erfc y/sqrt(4*v*t) (1 - erf = complementary error function)

Complementary error function is monotone decreasing function that goes from 1 to 0 and drops to 0.01 when η is about 2. Use this fact to define boundary layer thickness, ઠ as that distance y for which vx has dropped to a value of 0.01v0.

ઠ = 4sqrt(*v*t) is the natural length scale for diffusion of momentum.

Example 4.1-2: Re-solve preceeding illustrative example, but with fixed wall at distance b from the moving wall at y = 0. Flow system has steady state limit as t ⇾ inf. (previous did not).

∂vX/∂t = *v* ∂2vx/∂y2

I.C.: t <= 0, vx = 0 for all y

B.C.1: y = 0, vx = v0, t > 0

B.C.2: y = 2, vx = 0, t > 0

Introduce dimensionless variables

Փ = vx/v0; η = y/b; τ = *v*t/b2 (all values range from 0 to 1).

∂Փ/∂τ =∂2Փ/∂η2

Փ = 0 at τ = 0

Փ = 1 at η = 0

Փ = 0 at η = 1

τ = inf. 0 = ∂2Փinf/∂η2 with Փinf = 1 at η = 0, Փinf = 0 at η = 1

Փinf = 1 - η

Փ = (η,τ) = Փinf(η) - Փt(η,τ)

∂Փt/∂τ =∂2Փt/∂η2

Փt = Փinf at τ = 0

Փt = 0 at η = 0

Փt = 0 at η = 1

Փt = f(η)g(τ)

Substitute into differential

1/g (dg/dτ) = 1/f d2f/dη2

Designate constant as -c2 (who fukin knows y)

dg/dτ = -c2g

d2f/dη2 + c2f = 0

g = A3e-c2t

F = B sin(cη) + C cos(cη)

Apply boundary conditions

cn = n\*pi where n = 0, +/-1, +/-2, ...

Fn = Bnsin(n\*pi\*η)

Gn = Ane-n2pi2t

Փt = sum(Dne-n2pi2tsin(n\*pi\*η))

Multiply both sides by sin(mpiη) and integrate from n = 0 to n = 1 to solve for D